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SOME PREDICTED CLIMATIC EFFECTS OF A SIMULATED SAHARA LAKE

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10. ABSTRACT

Results of simulating the addition of a large body of water to the Sahara Desert using the Rand version of the Mintz-Arakawa general circulation model. The simulated lake is a relatively small perturbation to the earth's boundary condition and was chosen to test the hypothesis that an observable and statistically significant change would be produced in the local circulation. The flux and flux divergence of water vapor over North Africa was studied to determine where the water added to the atmosphere by the lake would be precipitated. A significant change in precipitation was subsequently observed over the selected area. A new statistical measure is described and used to measure the significance of the observed change.

11. KEY WORDS

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Some Predicted Climatic Effects of a Simulated Sahara Lake

R. R. Rapp and M. Warshaw

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PREFACE

The ability to discern small changes in the general atmospheric circulation as simulated by a numerical model is crucial to the study of purposeful or inadvertent climatic change. Past work (Warshaw and Rapp, 1973) has demonstrated that *very large* changes in surface boundary conditions have a significant effect in 60 days. This study is an extension of that investigation wherein only a *very small* modification (six computing grid points) is made to form a lake in the Sahara desert. A new statistical method is introduced and used to measure the significance of observed change in the mean precipitation and wind direction in the neighborhood of the simulated lake.

This report is one of a series on the dynamics of climate, a project sponsored by the Defense Advanced Research Projects Agency.

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SUMMARY

This report presents the results of simulating the addition of a large body of water to the Sahara desert using the RAND version of the Mintz-Arakawa general circulation model. The "Sahara lake" was chosen as a relatively small perturbation to the earth's boundary condition which the authors felt might produce an observable (and statistically significant) change in some property of the local circulation and short-term climate.

By observing the moisture flux and moisture flux divergence as a function of the lake addition, a region was selected in which to test the hypothesis that the precipitation was altered. A significant change in precipitation was subsequently observed over the chosen area. Similarly, we found a significant wind direction shift in the lower (800-mb) level of the model.

A new statistical procedure requiring only one control and one experimental run is described. The method achieves its utility by estimating a first-order Markov process from the sequential values of a variable, transforming away the serial correlation of the samples, then treating the transformed values as independent, stationary values.

Speculation on how these simulated results might be used to predict the actual change in the climate is also given.

ACKNOWLEDGMENTS

The authors are indebted to W. L. Gates and L. R. Koenig, both of The Rand Corporation, for their comments and careful review of this report.

CONTENTS

I. INTRODUCTION

The general circulation of the earth's atmosphere is controlled largely by the equator-to-pole temperature gradients, which in turn are caused by the differential in solar heating. The climate of a particular place is a function of its position relative to this meridional temperature gradient and also its position with respect to oceans and mountains.

Defining "climate" has proven elusive; we will paraphrase and adopt Leith's (1973)* comments on this subject by mentally dividing the land-ocean-atmosphere into two parts: First, the "internal system"--that part which is subject to relatively rapid fluctuation (weeks or less). Second, the "external system"--including the ocean--that changes slowly and influences the state of the internal system. Leith goes on to define climate in terms of averages over an ensemble of internal states which is nearly in equilibrium with the slowly changing external influences. Ensemble averages may change in response to changing external conditions in a finite time; thus we are able to speak of climate change.

Weather, on the other hand, has a variability that is essentially unpredictable beyond a few weeks. Lorenz (1969) has shown that the sequence of weather events is, to a large extent, dependent on the initial condition rather than on the thermal-gradient forcing function. In recent years, an effort has been made to determine the effect on the climate of purposeful or inadvertent changes in the earth's surface or content of the atmosphere. Faced with the large fluctuations of natural weather, it is difficult to assess how much of an observed change is actually caused by a change in the external system and how much is just the natural variation of weather.

Here and in an earlier paper (Warshaw and Rapp, 1973), the authors have inquired whether some change in the earth's surface would produce a change in the climate. Climatic change is operationally defined

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*Unpublished paper.

as a significant change in the 30-day mean of some meteorological variable of interest. Using a simulation of the global atmospheric circulation (the Rand version of the Mintz-Arakawa model; see Gates, et al., 1971), a model which appears to produce a fairly good representation of long-term averages (Gates, 1973a, 1973b), we previously looked for significant changes after subjecting the system to a massive change in surface boundary condition--namely, the removal of all the Arctic sea ice. Such changes were observed and subsequently reported (Warshaw and Rapp, 1973). We now turn our attention to a very small perturbation of the earth's surface, but with the same problem in mind--that of identifying statistically significant climatic change in the presence of natural weather fluctuations.

We place considerable importance in this report on the statistical analysis of results. Yet it is helpful to remember that the use of statistics to assess the relation between cause and effect is only a tribute to our ignorance of the underlying physical process. The global circulation model is, in effect, a complex hypothesis of the nature of the atmospheric circulation. To the extent that this hypothesis predicts the evolution of the circulation, given boundary and initial conditions, it can be used to infer short-term effects caused by changes in these conditions. If we further assume that changes in the atmospheric forcing functions are not modified by nature's feedback mechanisms (since these mechanisms are not present in the model), the simulation may be used to predict climatic change.

Even with these limitations, the system is too complex for us to be aware of all the interactions even on a relatively crude computing grid--to say nothing of the real world. That which we are not aware of, or cannot account for, we call noise, or randomness, and this in turn leads us to statistical and probabilistic methods to quantify our computed results.

The role of the physical scientist has not changed, however. He still performs an experiment on the global circulation by modifying some boundary condition (via the simulation, of course). And he still should have some hypothesis about climatic change. The legitimate question to ask the statistician is, "Is the observed difference in

such-and-such a variable attributable to my purposeful change in boundary condition, or is it just a random fluctuation?" The responsibility for explaining cause and effect still belongs to the investigator. To the question, "Was there any change, anywhere, that might be attributed to something other than random fluctuations?", statistical inference will not yield a satisfactorily sharp answer because of the large number of computing grid points and shortness of record length. We urge, therefore, that the statistics displayed in this report be viewed as evidence corroborating a prior theory of physical change--not a unique link connecting boundary condition change to computed result.

Section II describes the experiment--the addition of a very large lake in the Sahara desert. This experiment may not be reasonable in a political or economic sense, but it is possible in a technical sense. The physical reasoning leading to the experiment, the way the experiment was carried out, and the derivation of some statistical hypotheses are described.

Section III presents the statistical results of the study, but defers the description of technique to the Appendix. This technique consists of deriving adequate data from only one control simulation and one simulation with the addition of the lake. This is a marked change from earlier work which required a total of six simulations before the problem was statistically tractable.

Finally, in Sec. IV we treat briefly the relation between results predicted by a simulation and real climatological data. This is a topic of considerable importance and will receive increasing attention in future work.

II. THE LAKE SAHARA EXPERIMENT

A RATIONALE FOR THE LAKE

Studies of the effect of large lakes and irrigated tracts, done mainly in the Soviet Union (Nikalin, 1960), have indicated that, for distances a few kilometers away from the lake or the irrigation area, changes in the climate can be measured. The fallacy of expecting the creation of a lake in a desert region to increase the rainfall in the immediate region was expounded by McDonald (1962). He pointed out the need for a mechanism to produce the vertical motion required to condense water vapor and initiate precipitation. He also noted the large quantities of water contained in the atmosphere over desert regions. Lufkin (1959) observed that some desert regions appear to be sources of water to the atmosphere. Hence, there is no a priori reason to assume large changes in precipitation near such a lake.

There are, however, other reasons to study the effect of such a lake. It might have measurable effects on the radiation balance of the region--producing, in turn, changes in the local circulation. The large difference in evaporation between a desert and a lake will undoubtedly affect the rainfall somewhere. The question of where the lake effect will be felt is perhaps the most cogent reason to perform the experiment.

In some of the popular Soviet literature (Rusin and Flit, 1962), the creation of rather large inland seas is discussed and, although no engineering plans are presented, some feasibility estimates have been made. One such proposal was an increase in the size of Lake Chad in the south-central Sahara region. Here an inland sea about the size of the Caspian Sea would be formed. We call this body of water the Sahara Lake and will simulate its effects with the Rand version of the Mintz-Arakawa general circulation model.

The original suggestion for forming this lake was to dam the Congo River at the Stanley Narrow, where the Congo cuts through the coastal range on its way to the Atlantic. This would form a huge sea in the Congo flooding most of the Congo Basin, and the water would then, by

process of digging canals or tunnels, be fed across a low, narrow ridge into the Chari River, which feeds Lake Chad. The large additional flow of water from the Congo and Ubangi Rivers into Lake Chad would cause it to grow and in a matter of ten to fifteen years form a lake of approximately 1.4 x 10^6 sq km in the southern Sahara.

We looked at the maps and topographies of Africa and concluded that without flooding the Congo Basin, one could dam and divert the Ubangi River into the Chari with a quite technologically feasible engineering program. Certainly a program no greater than the Soviet diversion of the Pechora River into the Volga and the Ural Rivers. Since the drainage basin of the Ubangi is on the equatorial side of the range of mountains that separates the Chari Basin from the Ubangi Basin, its drainage area is larger and receives an annual rainfall two to three times that which falls over the Chari Basin. We made no specific calculations on how long it would take this diverted Ubangi to create a large Sahara lake, but believe it would be of the same order of magnitude as the more difficult and wasteful formation of a large lake in the Central Congo.

DESCRIPTION OF THE LAKE

Further study of the lake formation was not pursued. Ideally, the additional flow of water should be balanced against the additional evaporation to make a quantitative determination of the size of the lake at stabilization. Unfortunately, data on the rate of flow of the Chari and the Ubangi could not be found. We assumed, therefore, that the lake would fill to approximately the 400-meter level. Since the Global Circulation Model (GCM) has a resolution of 4° of latitude and 5° of longitude, it was not possible to make a realistic fit to the actual topography. The final configuration of the lake as it was modeled is shown in Fig. 1. It should be noted that the alteration required to insert the lake into the model affected only six grid squares. This amounts to less than 0.2 percent of the earth's surface, but over a million square kilometers.

The Rand version of the Mintz-Arakawa model requires that the temperature of water surfaces be specified and remain constant. The water

North Latitude (deg) \ East Longitude (deg)	10	15	20
18	17.5°C	Land	17.5°C
14	21.3°C	21.9°C	24.8°C
10	Land	25.3°C	Land

Fig. 1 — Location, size, and surface temperature of the Sahara lake

surface of the lake is maintained at the mean surface air temperature of the region where the mean is taken over the time period used in the simulation. The values in each square of Fig. 1 are the assigned temperatures.

DEVELOPMENT OF HYPOTHESES

The model with the lake was run for 60 days--a period simulating conditions from the end of December to the end of February. The model without the lake had been run previously for this same period. As noted earlier, we expected little change in precipitation surrounding the lake. In order to produce a testable hypothesis, we traced the flux of water vapor from the lake to some point where enhanced precipitation would be expected. To this end, the moisture flux and moisture flux divergence over North Africa was computed for both the control and the experiment.

In the Mintz-Arakawa model, moisture flux is computed every simulated half hour, but the values are not retained as part of the history record. The values, however, may be reconstructed from available wind,

density, and specific humidity values which are saved on the record every six simulated hours. Moisture flux is given by $\bar{F} = \rho q \bar{V}$, where ρ = air density, q = specific humidity, and $\bar{V}$ = wind velocity. The scheme for adapting this continuous representation to a discrete computing grid is given by Gates, et al., 1971. The moisture flux divergence, div $\bar{F}$, is computed in a manner identical to the Mintz-Arakawa model.

Figures 2 and 3 show the mean direction of the moisture flux and the magnitude of the flux divergence for the control run for January (first 30 days) and February (second 30 days) respectively. Figure 2 shows a weak flux of moisture from the North Atlantic and the Mediterranean from north and south across the Sahara. Weak divergence is present over most of the Sahara. South of the Sahara, in the Ubangi and Congo Basins, this weak flux is joined by flows of moisture from the Indian Ocean and the South Atlantic to produce a region of convergence centered near 6°S, 25°E. In February (Fig. 3), the pattern is very similar, but very weak convergence is present over the Sahara. The center of convergence over the Congo Basin has shifted slightly toward the northwest and the area has expanded toward the northwest. The northwestward shift of the convergence zone is compatible with previous studies of the Intertropical Convergence Zone (ICZ) over Africa (Schutz, 1973*). In January, it has reached its southernmost position and in February it has started to shift northward.

Figure 4 shows the same January parameters as Fig. 2, but with the lake inserted. The direction of the flux shows very little change from the control, but the large divergence due to evaporation from the lake dominates the picture. The convergence over the Red Sea seems to have increased, and the central African convergence appears to have extended to the north and east. It must be recalled that the first fifteen days of the experiment are strongly influenced by initial conditions which were calculated without the presence of the lake (Warshaw and Rapp, 1972); thus Fig. 4 represents only a portion of the effect which would be expected had the lake been inserted early in December.

Figure 5 shows the situation in February. The effects of the

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*Personal communication from C. Schutz, The Rand Corporation.

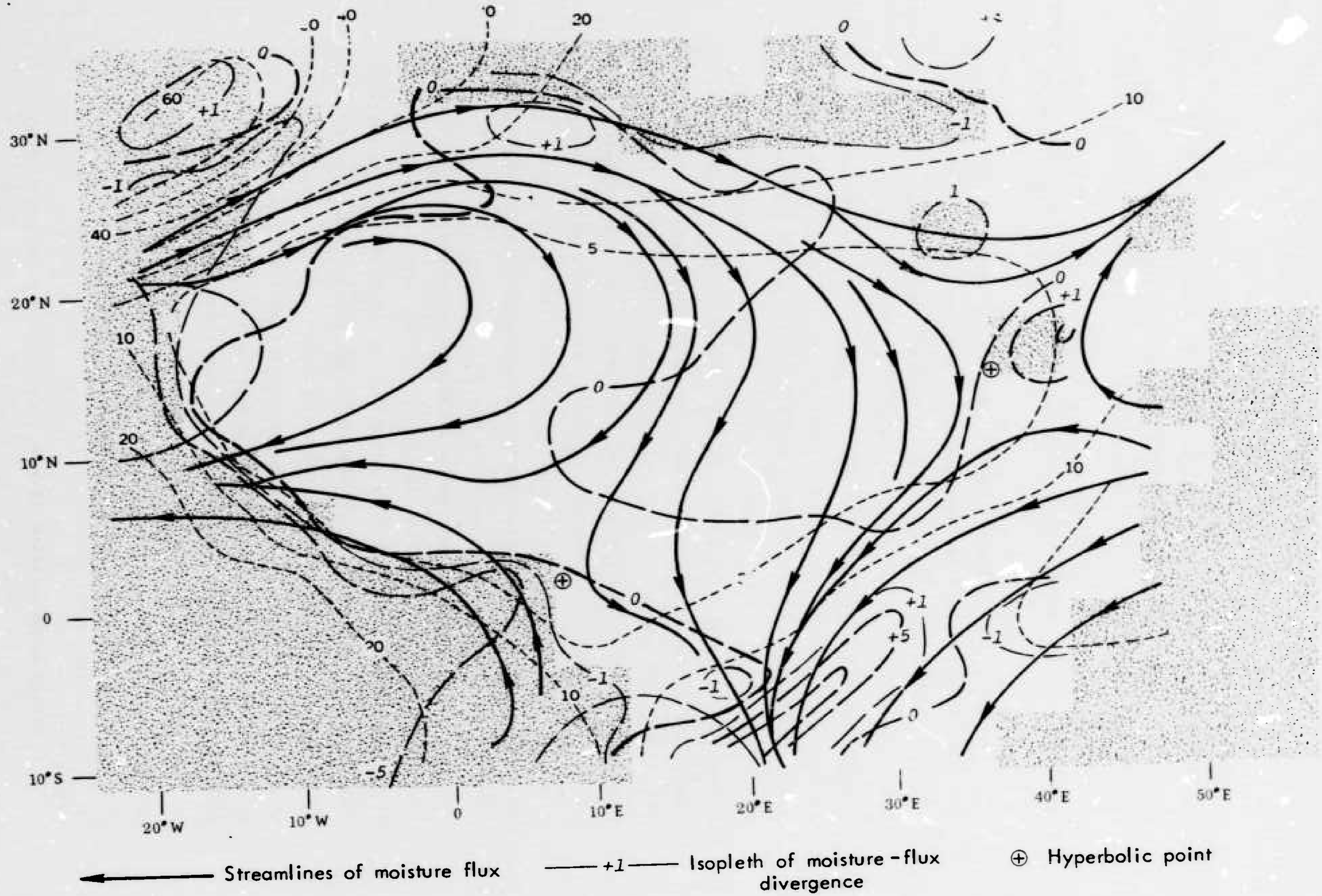


Fig. 2 — The mean moisture flux and the meon moisture-flux divergence over the northern part of Africa, as calculated by the model for o control run representing Jonuory conditions

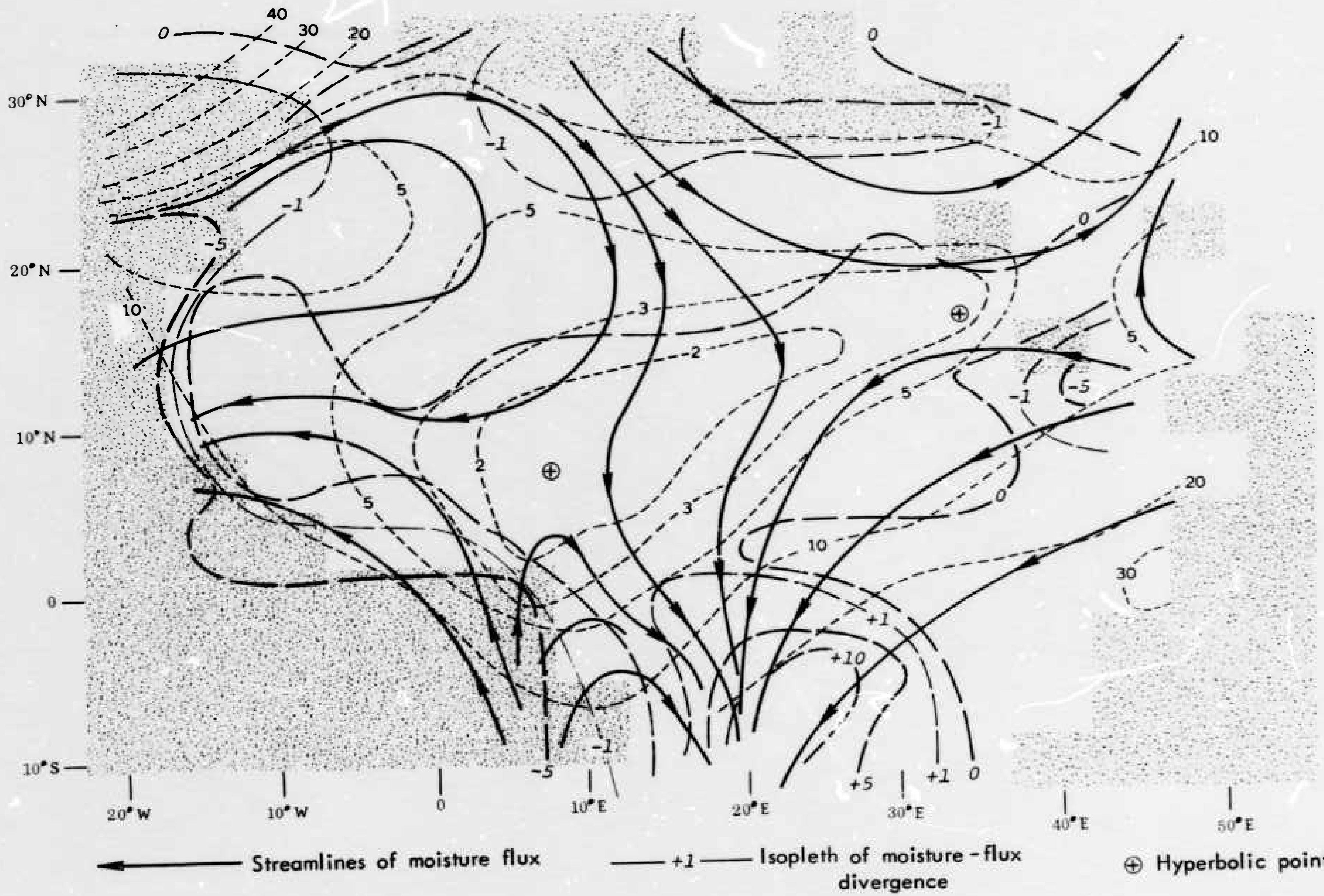


Fig. 3 — The mean moisture flux and the mean moisture-flux divergence over the northern part of Africa, as calculated by the model for a control run representing February conditions

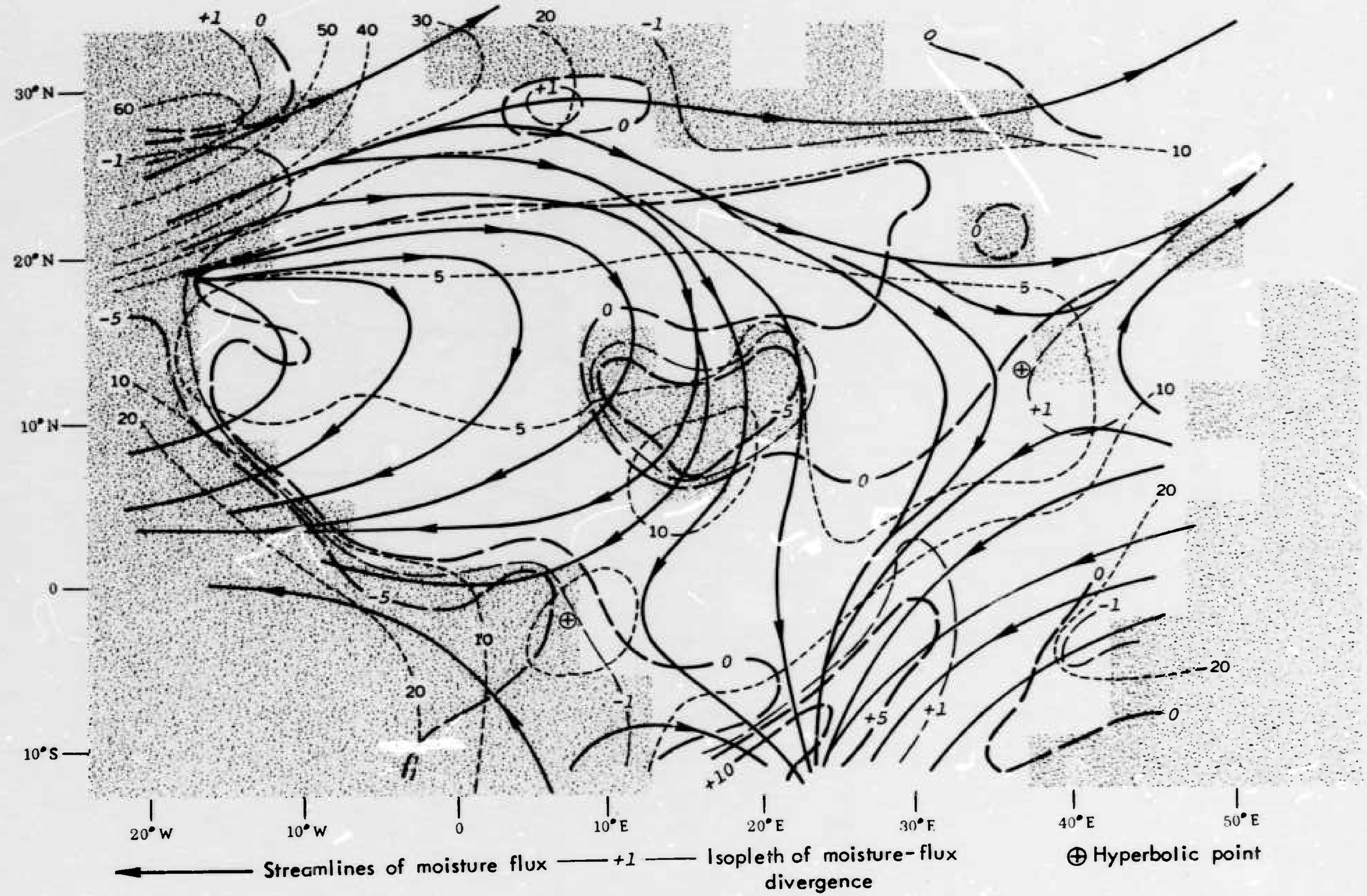
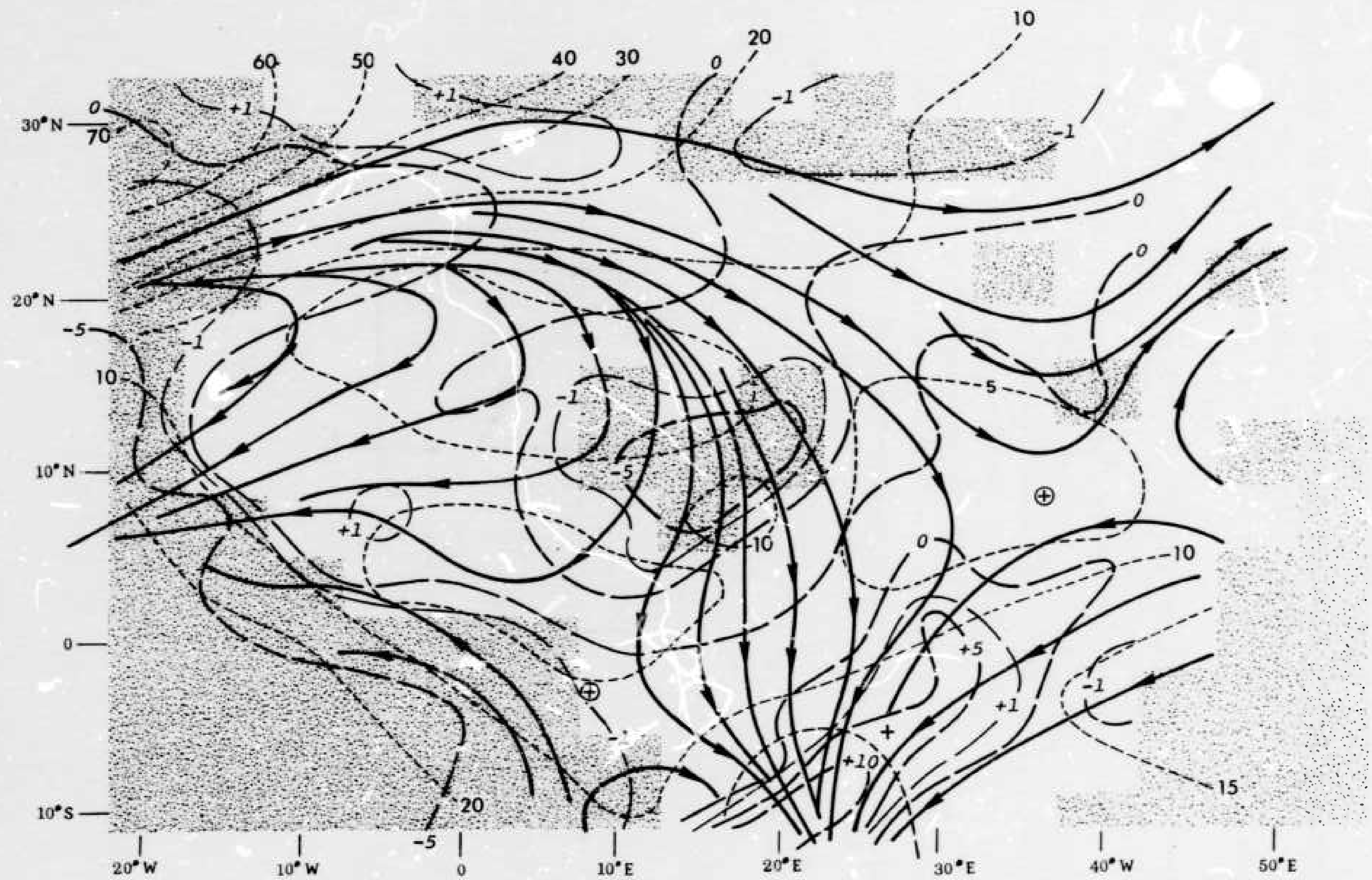


Fig. 4 — The mean moisture flux and the mean moisture-flux divergence over the northern part of Africa, as calculated by the model for the experimental run for January



Streamlines of moisture flux —— +1 —— Isopleth of moisture-flux divergence ⊕ Hyperbolic point

Fig. 5 — The mean moisture flux and the mean moisture-flux divergence over the northern part of Africa, as calculated by the model for the experimental run for February

initial conditions have largely disappeared and the patterns of flux and flux divergence appear to have changed under the influence of the lake. The flux of water vapor from the north has been enhanced and the flux from the South Atlantic has been weakened. This change has prevented the growth of the convergence zone to the northwest and enhanced the convergence over the mountains of east equatorial Africa.

The changes in flux and flux divergence suggest a testable hypothesis for a change in the rainfall pattern. Grid points at (25°E, 2°N), (25°E, 2°S), and (30°E, 2°N) appear to have been changed from convergent to divergent moisture flux divergence, and grid points (30°E, 2°S), (35°E, 2°N), and (35°E, 2°S) appear to have been changed from divergent to convergent moisture flux divergence (see Fig. 6). Our first hypothesis is, on the basis of the above reasoning, *that the addition of the lake changed the precipitation pattern of equatorial Africa so that the first set of points received less rain and the second set received more rain.*

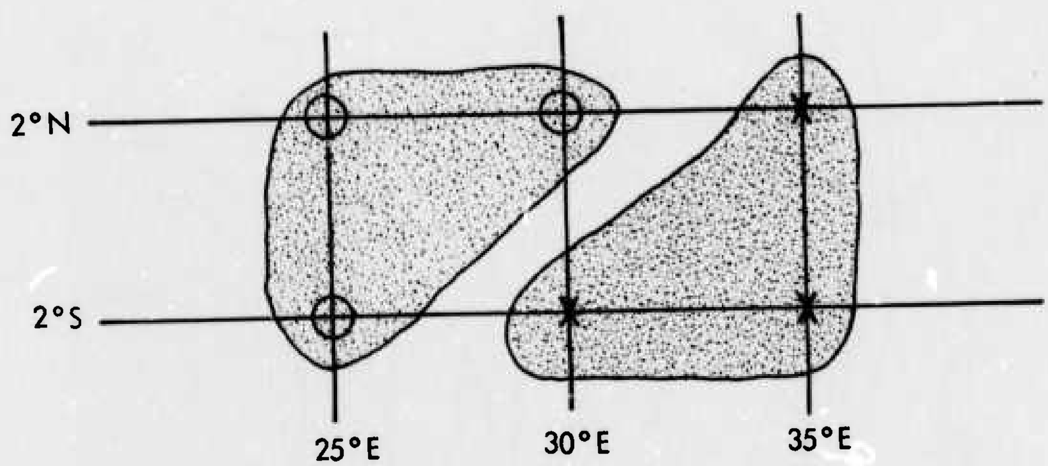


Fig.6 — Mintz-Arakawa grid points used to test the hypothesis of change of precipitation pattern

The flux and flux-divergence maps suggest that, in addition to increasing the water vapor, the lake produced a change in the lower-level winds. Perusal of the surface temperature maps indicates that the mean temperature over the lake was slightly higher than the temperature in the control. There was also a slight increase in the vertical velocity in the area after the lake was inserted. From this, we deduce that a weak thermal low developed over the lake despite the assignment of the observed mean temperatures to the lake's surface. This should have the effect of adding a small westerly component to the wind south of the lake. We therefore propose a second hypothesis *that the mean wind in the lower level for six grid points south of the lake shifted from northeasterly to northerly*. The method for testing these hypotheses and the outcome of the test are described in the next section.

III. STATISTICAL ANALYSIS OF THE EXPERIMENT

STATISTICAL TECHNIQUES

The question is whether our hypotheses are acceptable, that is, can we conclude that the addition of the lake *caused* the observed change? The topic of significant change and the discrimination of such change in the presence of natural variability has been treated earlier (Warshaw and Rapp, 1972). In that report, differences in mean values were sensed through an analysis-of-variance test requiring at least three replications of both the "control" and "experimental" computer runs. Because these runs are very costly in terms of computer time, we sought for a way of extracting essentially the same information from only one control and one experimental run.

This requires the extraction of an uncorrelated subsequence from the highly correlated sequence of simulated meteorological data. Given such a subsequence of multivariate data from both control and experimental runs, it is possible to estimate the required covariance matrices and utilize the uncorrelated data in a multivariate hypothesis test of the equality of time averaged variables. The statistical procedures and their embodiment in an interactive computer program have been reported by Warshaw (1973) and the reader is referred there for details of the method. An outline of the methods is given in the Appendix.

TEST RESULTS

Precipitation. Our first hypothesis, that the addition of the lake changed the precipitation pattern of equatorial Africa so that the left set of points received less rain and the right set received more rain, is illustrated in Fig. 6. The variables are first averaged over each day to eliminate the diurnal periodicity. Then the three points marked ⊕ are averaged together to form a left region and the three points denoted ⋇ are averaged to form a right region. We now have a 60-day sample consisting of two-component vectors representing the left and right values of each sample.

The precipitation averaged over the last 30 days of the run for the left and for the right region for both the control and the Lake Sahara experiment is given in Table 1.

Table 1

PRECIPITATION
(mm/day)

Region	Control	Lake Sahara Experiment
Left	52.8	21.1
Right	1.4	40.0

If all six points are summed, there is no significant difference between the total precipitation for control and that for the experiment. However, when the multivariate structure is maintained by separating left and right, we reject the hypothesis of equality of mean precipitation at the 0.025 significance level. Achieving this result required that we utilize the Markov-filter approach, then thin the resulting transformed data by using only every third sample.

Winds. Our second hypothesis, *that the mean wind in the lower level for six grid points south of the lake shifted from northeasterly to northerly*, is illustrated in Fig. 7. Using the same techniques used

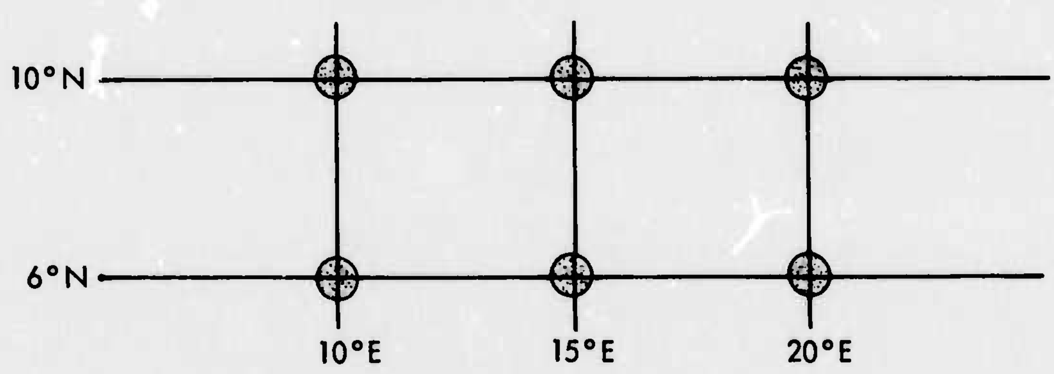


Fig.7 — Mintz-Arakawa grid points used to test the hypothesis on wind shift

in testing for differences in precipitation, we form a two-component vector made up of the zonal component, U, and meridional component, V, of the wind. A single sample of these winds consists of the value obtained by averaging over all six grid points and four values for each day. The 30-day mean values for the wind at $\sigma = 0.75$* are shown graphically in Fig. 8.

The addition of the lake has rotated the wind counterclockwise, and this effect is significant at the 0.05 level. There was no significant change in the magnitude of the wind.

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*The vertical coordinate in the Mintz-Arakawa model is the sigma surface, σ, defined as $\sigma = (P-P_T)/(P_s - P_T)$ where P_s = surface pressure and P_T = pressure at top of modeled atmosphere (P_T = 200 mb). Note that $0 \leq \sigma \leq 1$.

IV. MODEL RESULTS VERSUS OBSERVED DATA

This section is an initial speculation on how simulated results (and simulated differences) might be used to predict real-world differences in light of the fact that results of the simulation do not agree exactly with real observations.

The previous section has indicated that, within the context of the model, the insertion of the lake did indeed produce a change in the rainfall patterns of equatorial Africa. However, since the model still differs from observations of the real world, we need first to determine how much and in what way the *observed* rainfall around our six points differs from that of the model's control climate. Then, assuming that the alternative future climate will differ similarly from the climate of the model experiment, we can estimate by analogy what rainfall the alternative future would bring.

Figure 9 is a schematic of the comparisons and the inferences we would like to make. The bottom two boxes represent the analysis presented above showing the differences between the control run and the experiment. The development and testing of the statistical hypotheses lend credence to the concept that the lake caused the differences in precipitation. The left-hand boxes represent the relationship between the model control and the observed contemporary climate. These first two relationships are measurable; the remaining two, of course, are inferred. The right-hand boxes are the two analogous futures, and *by assumption*, relate to one another approximately as do the two left-hand boxes. The top two boxes--the observed real rainfall and the alternative future rainfall--exhibit differences that are analogous to--and can be inferred from--the differences between the two lower boxes.

Figure 10 shows this schematic filled in with isohyets based on ten-year February means from World Weather Records (U.S. Department of Commerce, 1967) and total monthly precipitation from the control and the Sahara Lake experiment. Comparing the observed and control patterns, it appears that the maximum in the control is too high but is located in the right place. The patterns of the lines match as well

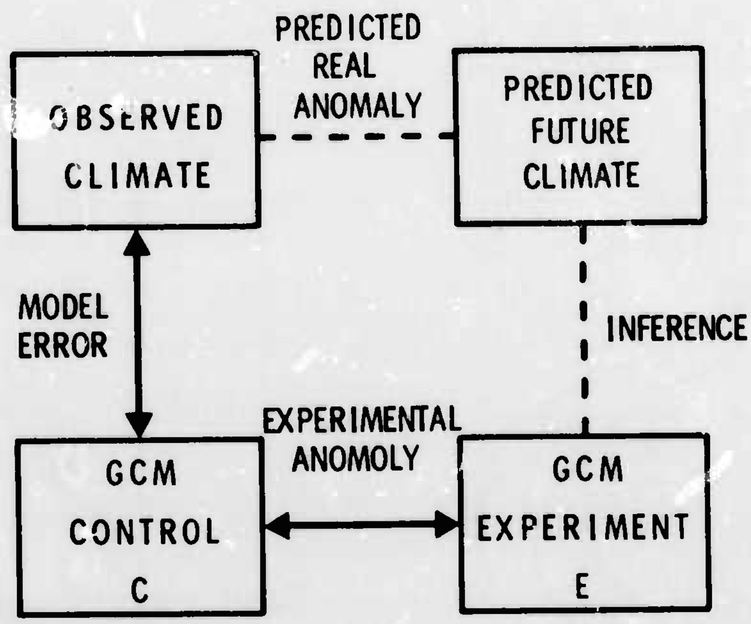

Fig. 9 — Schematic of comparisons and inferences

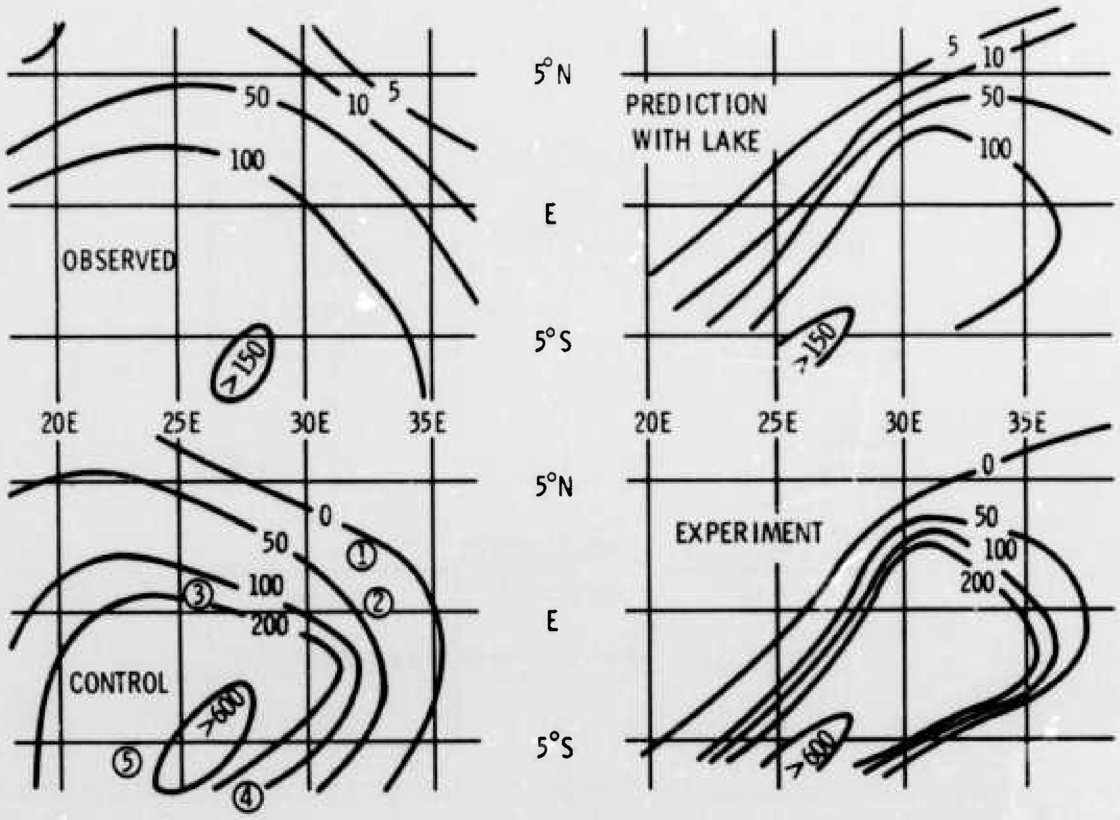

Fig. 10 — Patterns of February total rainfall

as might be expected when one month is compared with a ten-year mean. We know from previous results (Gates, 1972a) that the model tends to produce too much rain in the rainy regions. But those comparisons were based on a single month's run compared to long-term mean values. Such comparisons are useful to calibrate the model, but in order to answer questions about potential climatic change a long-term mean is insufficient; we need to look more closely at the interannual variation of the quantities under discussion. In order to compare the data from the control with the real world, we chose five stations in equatorial Africa from World Weather Records for which ten years of data were available.

The distribution of monthly rainfall amounts was fitted with an incomplete gamma function of the form:

$$f(x) = \frac{1}{\beta^{\gamma}\Gamma(\gamma)} x^{\gamma-1} e^{-x/\beta} \qquad (5)$$

The mean of this distribution is $\beta\gamma$ and the standard deviation is $\beta\sqrt{\gamma}$. Figure 11 shows the cumulative distributions for the five chosen stations, which are indicated by the circles on Fig. 11. From Fig. 12 we read the precipitation limits for the interquartile range.* These values, together with the computed control rainfall values as interpolated from Fig. 10, are shown in Table 3. Stations 1, 2, and 4--all on the eastern side of the maximum--fall well within the interquartile range. Station 3, Kisangani, which is north of the maximum, is high, but the value of 197 mm/mo would be expected to be exceeded in one year out of 11. Station 5, Luluabourg, was computed to have far too much rain. The probability of getting 550 mm is much less than 1 percent.

In order to correct for this model deficiency in estimating the effect of the lake, we made a very crude scaling of the precipitation. Figure 12 is a plot of observed versus computed rainfall with the mean and interquartile range of the five stations plotted. A smooth, freehand curve was drawn through the mean values of Gulu and Kampala and

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*The interquartile limits define a range so that 25 percent of the values lie below the lower limit, 50 percent between the limits, and 25 percent above the upper limit.

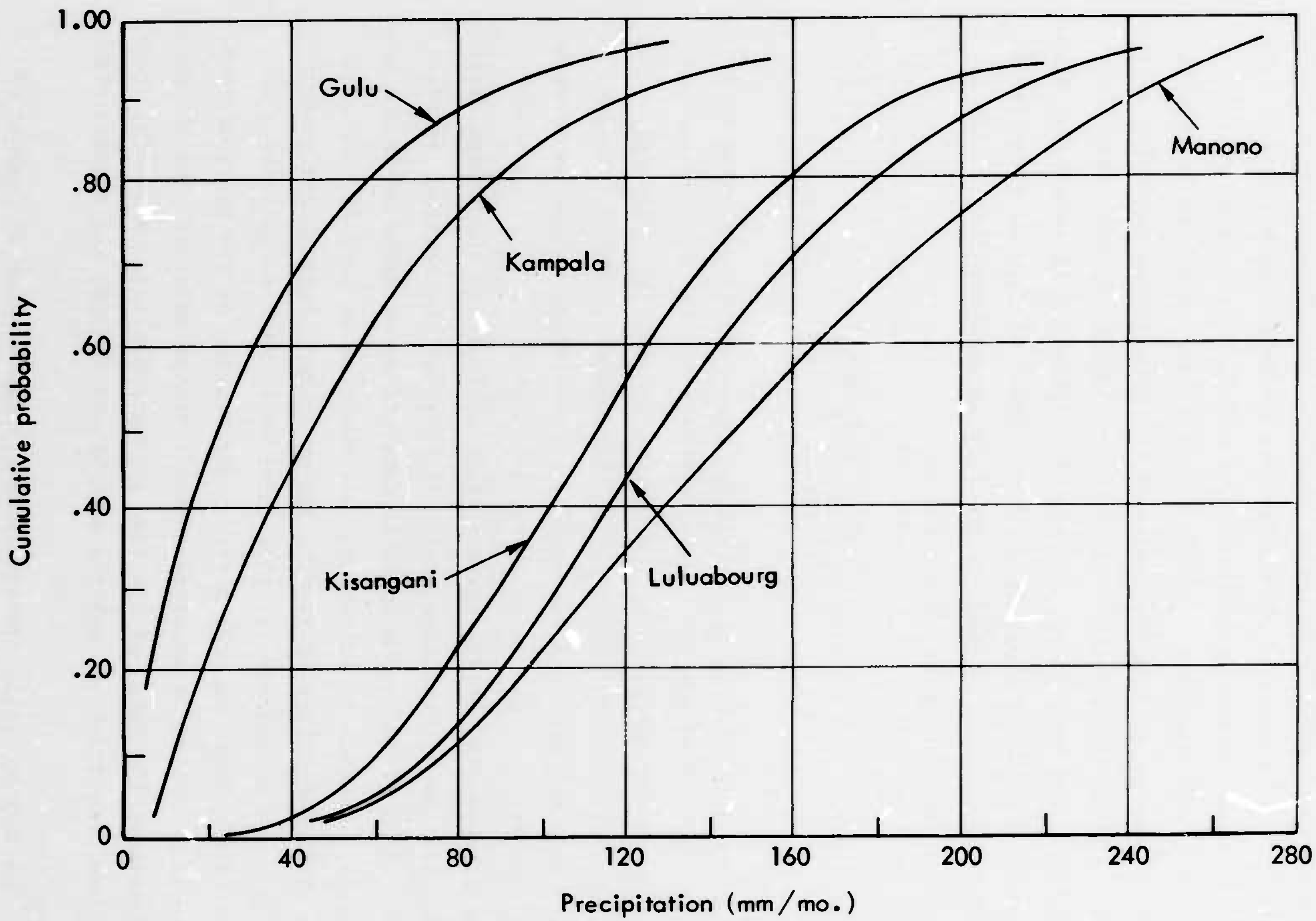

Fig. 11 — Gamma distribution fitted to rainfall data for five stations in equatorial Africa

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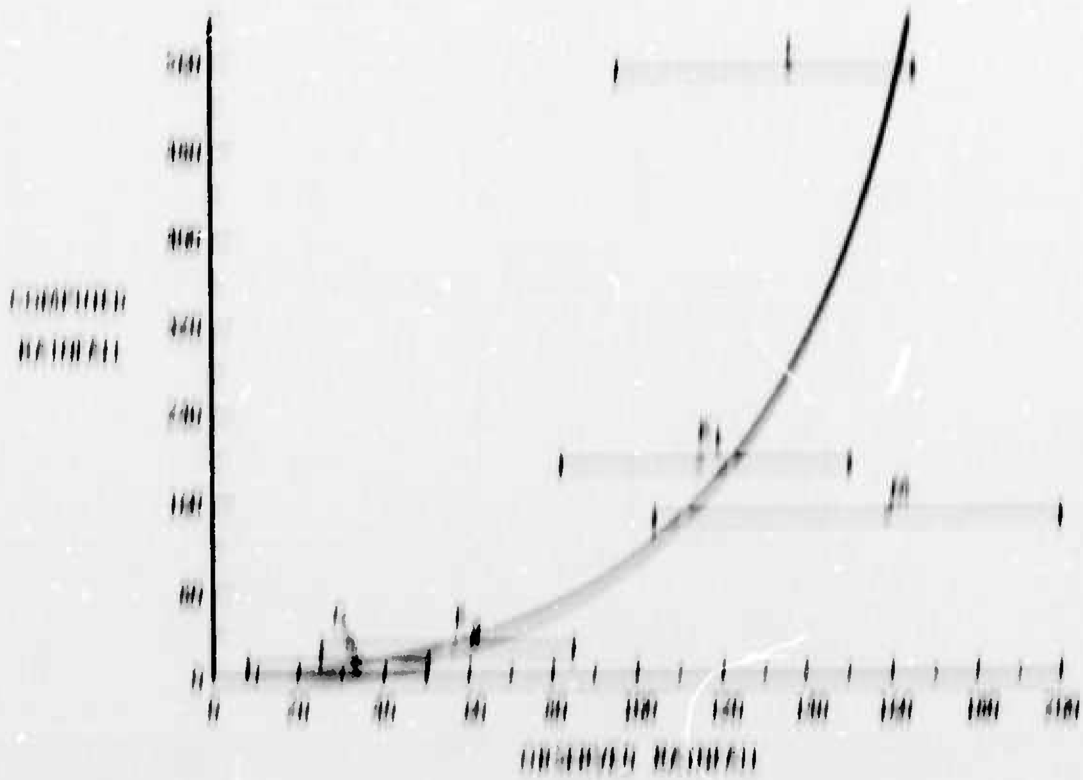

Fig. 14 — Scaling curve to adjust computed rainfall to approximate observed rainfall

fitted within the bars that include the interquartile range. This curve was then used to scale the isohyets of the experiment to produce the predicted rainfall pattern with the lake shown in the upper right-hand portion of Fig. 10.

Table 3

CONTROL RAINFALL AND INTERQUARTILE LIMITS
OF OBSERVED RAINFALL

Station				
No. in Fig. 11	Name	Computed Rainfall	25% Limit	75% Limit
1.	Gulu	12	8	50
2.	Kampala	30	25	85
3.	Kisangani	197	81	147
4.	Manono	150	104	200
5.	Luluabourg	550	96	166

V. CONCLUSIONS

We have attempted to predict, by the use of a numerical model, the effect of introducing a large body of water into a desert region. In order to strengthen the hypothesis that the observed changes were not simply due to the normal variation of the atmosphere--or the model--it was necessary to utilize statistical methods of analysis. Such analysis can never prove a hypothesis, but it can protect us against claiming a causal change when in fact there was none except that due to normal variations. The change in rainfall pattern, according to our analysis, had less than a 0.05 probability of occurring given the basic variability of the model. We, therefore, conclude that the difference in the mean value over the last 30 days between the control and the experiment indicates the nature of the change which a lake of the specified location, size, and temperature of our hypothetical lake might have on the precipitation.

The pattern of changes is not inconsistent with previous qualitative arguments: McDonald (1962) presented a strong case for the need for a precipitation mechanism to release any water added to the atmosphere and Lufkin (1954) presented evidence that arid regions can actually be a source of atmospheric moisture. Thus, the fact that the increased precipitation, except for two isolated showers over the lake, occurred in mountainous regions some 900 km distant is quite reasonable.

The inadequacy of the model in replicating the atmosphere must be recognized. We have resorted to a rough scaling procedure to eliminate the overprediction of rainfall. It would be more satisfactory if we did not need any type of correction, but if we were to wait for models which are without flaw, we might never be able to make climatic predictions.

It must be borne in mind that the results presented here represent a change brought about by a very small change in the surface features of the planet. Moreover, these changes are changes from an assumed state of the surface. We believe that we have demonstrated one major effect of the lake in the absence of other surface modifications, but

we can say nothing about other forces which may produce similar results. Changes in the temperature of the sea surface in the equatorial Atlantic, for example, could have a profound effect on African rainfall, but these have been assumed to remain unchanged between the control and the experiment.

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Appendix

STATISTICAL ANALYSIS OF SIMULATED CLIMATIC CHANGE

Assume two discrete finite stochastic sequences $\{x_i\}$ and $\{y_i\}$, where $i = 1, 2, \ldots, n$, where x_i and y_i are both p-tuple vector variables. The sequences $\{x_i\}$ and $\{y_i\}$ represent the values produced by two runs of the general circulation model. In general, the elements of the sequence $\{x_i\}$ are not independent, nor are the elements of $\{y_i\}$. The p elements of the vectors x_i and y_i also have unknown and not necessarily equal covariance matrices. These two sequences constitute the sample data for all the tests which follow.

PERIODIC COMPONENTS

Experience has shown that strong periodic components in the data, e.g., the diurnal period, must be eliminated by either a Fourier analysis of the data followed by a subtraction of the estimated periodic component, or by averaging over the undesirable period. For the work reported here, we have data every six hours and choose to average every four samples to produce a new sequence of average daily variables.

INDEPENDENCE TESTS

The turning point test (Kendall, 1966), the rank correlation test (Kendall, 1966), and the Durbin-Watson test (Durbin and Watson, 1950, 1951; Malinraud, 1966) are applied to the data testing for the present periodicities, monotonic trends, and significant first-order correlation, respectively. These tests may be applied at any time and on subsequences of the original data (such as using only every k^{th} value) in a search for an uncorrelated sequence. A sequence which has no periodicities, no trends, and a negligible first-order correlation is not, of course, automatically independent, but we will take this as a strong indication of independence.

CORRELATION COEFFICIENTS AND SPECTRAL ANALYSIS

Two other measures of independence are the values of the first k correlation coefficients and the power-density spectrum. If all k

correlation coefficients are much less than unity and if the discrete components of the power spectrum are approximately equal, the elements of sequence will be assumed independent.

A MARKOV CHAIN FILTER

For data derived from the Mintz-Arakawa simulation, it rarely occurs that the original sequence approaches independence. If, however, we have any a priori knowledge of the nature of the dependence, it is possible to construct a transformation which eliminates the dependence between samples. For instance, if a sequence were generated by a first-order Markov process, $x_{i+1} = \rho_1 x_1 + \varepsilon_{i+1}$, where $0 \leq \rho_1 \leq 1$ and $\{\varepsilon_i\}$ is a sequence of independent normal random variables, the dependence between successive elements may be removed by the transformation

$$z_{i+1} = x_{i+1} - \rho_1 x_i, \quad i = 1, 2, \ldots, n - 1. \tag{1}$$

There is no reason to believe that our $\{x_i\}$ *is* a first-order Markov process and that the transformation will help. But we do know from experience that meteorological sequences tend to have monotonically decreasing correlation coefficients as a function of lag (if the diurnal periodicities are eliminated) and also, the *closer* the sequence is to Markovian, the more efficacious the transformation. We use this transformation extensively for Mintz-Arakawa data because it does tend to produce sequences with nearly uncorrelated elements.

TEST STATISTICS

In practice, we apply the Markov chain transformation to both the control and the experiment data. Next, we found it necessary to use every second, the sometimes every third, sample of transformed data to achieve independence. So the transformation was not quite good enough since the original data were not actually generated by a Markov process, but thinning the transformed data does produce the desired property.

Further experience has shown that the form of Hotelling's T^2-test that foregoes half of the degrees of freedom and does not require equal

covariances (Anderson, 1958) provides a sharper test. We compute the test statistic

$$T_u^2 = u(\bar{x} - \bar{y})'S_u^{-1}(\bar{x} - \bar{y}) \tag{2}$$

where n = sample size, $\bar{x},\bar{y}$ are multivariate means, and S_u, the estimated covariance matrix, is given by

$$S_u = \frac{1}{n - 1} \sum_{i=1}^{n} (x_i - y_i - \bar{x} + \bar{y})(x_i - y_i - \bar{x} + \bar{y})' \tag{3}$$

The critical region is

$$T_u^* = \frac{(n - 1)p}{n - p} F(p, n - p) \tag{4}$$

where p is the dimensionality of the multivariate samples and F(a,b) is the central F-distribution with (a,b) degrees of freedom for the numerator and denominator, respectively. We reject the hypothesis of equal means if $T_u > T_u^*$.

REFERENCES

Anderson, J. W. (1958): *An Introduction to Multivariate Analysis.* John Wiley & Sons, New York, 374 pp.

Durbin, J., and G. S. Watson (1950, 1951): "Testing for Serial Correlation in Least Squares Regression," *Biometrics*, December 1950 and June 1951.

Gates, W. L., E. S. Batten, A. B. Kahle, and A. B. Nelson (1971): *A Documentation of the Mintz-Arakawa Two-Level General Circulation Model*, The Rand Corporation, R-877-ARPA.

Gates, W. L. (1972): *The January Global Climate Simulated by the Two-Level Mintz-Arakawa Model: A Comparison with Observation*, The Rand Corporation, R-1005-ARPA.

Gates, W. L. (1973a): "Analysis of the Mean Forcing Fields Simulated by the Two-Level Mintz-Arakawa Atmospheric Model," *Monthly Wea. Rev.*, Vol. 101, pp. 412-425.

Gates W. L. (1973b): *The Simulation of January and July Climate with an Improved Two-Level General Circulation Model*, The Rand Corporation (in preparation).

Kendall, M. G. (1966): *The Advanced Theory of Statistics*, Vol. III, Hafner Publishing Co., New York, 552 pp.

Lorenz, E. N. (1969): "The Predictability of a Flow which Possesses Many Scales of Motion," *Tellus*, Vol. 21, pp. 289-307.

Lufkin, D. H. (1959): *Atmospheric Water Vapor Divergence and the Water Balance at the Earth's Surface*, Massachusetts Institute of Technology, Scientific Report No. 4, General Circulation Project.

Malinraud, E. (1966): *Statistical Methods of Econometrics*, North-Holland Publishing Co., Amsterdam, 631 pp.

McDonald, J. E. (1962): "The Evaporation-Precipitation Fallacy," *Weather*, Vol. 17, No. 5, pp. 168-177.

Nikalin, P. I. (1966): *Effects of Kuybyshev Reservoir on Microclimate*, Komsomol'skiy, Gidrometeorlogichskeya Observatoriya, Sbornik Rabot, No. 6, pp. 62-71.

Panofsky, H. A., and G. W. Biel (1958): *Some Applications of Statistics to Meteorology*, Pennsylvania State University, University Park, Pa.

Rusin, N. P., and L. A. Flit (1962): *Methods of Climate Control*, Moscow, USSR.

U.S. Department of Commerce (1967): *World Weather Records*, ESSA, U.S. Government Printing Office, Washington, D.C., 545 pp.

Warshaw, M. (1973): *The Statistical Analysis of Simulated Climatic Change*, The Rand Corporation, R-1298-DOT.

Warshaw, M., and R. R. Rapp (1973): "An Experiment on the Sensitivity of a Global Circulation Model," *J. Appl. Meteorol.*, Vol. 12, No. 1, pp. 43-49.